Sanitary and storm sewers are an integral part of urban infrastructure, and their adequate design has a direct bearing on such undesirable consequences as sewer overflows and flooding. The hydraulic design of sewers depends fundamentally on the accuracy with which the flow rate can be expressed as a function of the depth of flow in the sewer. The conventional approach to estimating the relationship between flow rate and flow depth in the United States is to use the Manning equation with a constant Manning’s n. This approach is valid if the following conditions are met: (1) the flow is fully turbulent; and (2) the condition for a constant Manning’s n is satisfied. In most cases of practical interest, these conditions are not met, the Manning equation is technically not applicable, and the Manning equation is used anyway. This article provides a clear delineation of the limitations of using the Manning equation to describe the hydraulics of sewer flows and provides an alternative formulation based on the Darcy-Weisbach equation.

**Theory**

Uniform flow in open channels under all flow regimes can be adequately described by the Darcy-Weisbach (DW) equation

$$Q = A \sqrt{f \frac{g}{R S_0}}$$

where $Q$ is the flow rate, $A$ is the flow area, $f$ is the friction factor, $R$ is the hydraulic radius, and $S_0$ is the slope. The friction factor, $f$, can be approximated by (ASCE, 1963)

$$f = 0.079 R^{2/3}$$

where $k_s$ is the equivalent sand roughness and $Re$ is the Reynolds number given by

$$Re = \frac{V}{u*}$$

where $V$ is the average velocity ($= Q/A$) and $u*$ is the kinematic viscosity of the fluid. Under turbulent flow conditions, $u* k_s > 70$ (Yang, 1996; Rubin and Atkinson, 2001) where $u* = (gR S_0)^{1/2}$ and hence the turbulent flow criterion for water ($u = 1.0 \times 10^{-6}$ m/s) can be expressed as (Chin, 2006)

$$k_s \sqrt{RS_0} > 2.2 \times 10^{-5}$$

Manning equation can be used in lieu of the Darcy-Weisbach equation, in which case,

$$Q = A R^{1/2} n \sqrt{RS_0}$$

where $n$ is the Manning roughness coefficient. Comparing the Manning equation with the Darcy-Weisbach equation for fully turbulent flow conditions, $(Re \rightarrow \infty)$ gives the following expression for the variation of $n$ as a function of hydraulic radius:

$$n = \frac{2}{k_s} \left( \frac{1}{S_0} \right)^{1/2}$$

(6)

The right-hand side of this equation remains approximately constant (± 5 percent) when

$$4 \leq \frac{k_s}{k} < 500$$

(7)

in which case (Sturm, 2001)

$$n = 0.039$$

(8)

The conventional method of estimating the flow rate in sewers as a function of the flow depth is using the Manning equation with constant $n$, which requires that both Equations 4 and 7 are satisfied. The Manning equation can be used with variable $n$ provided that Equation 4 is satisfied and the variability in $n$ as a function of flow depth is described by Equation 6. In contrast to these limitations on the Manning equation, the DW equation can be used to describe the flow rate as a function of depth of flow for all regimes.

**Analysis**

It is convenient to describe sewer hydraulics in terms of variables that are normalized relative to full-flow values. Denoting normalized quantities by asterisks (Equations 1 and 2) can be expressed in the form

$$Q_* = \frac{Q}{Q_{full}} \left( \frac{k_s}{k} \right) \left( \frac{1}{Re_{full}} \right)$$

(9)

where $Q_*$ is the flow rate normalized relative to the full-flow flow rate, $k_s$ is the full-flow relative roughness, and $Re$ is the full-flow Reynolds number such that

$$k_s = \frac{k_s}{k} \quad \text{and} \quad Re = \frac{k_s}{k}$$

(10)

Continued on page 56

**Hydraulics of Sewer Flows**

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where \( \theta \) is the apex angle of the triangle formed by the center of the pipe (apex) and the top width of the water surface (base). Combining Equations 9 and 11 gives the following relationship between the normalized flow rate, \( Q^*_M \), and apex angle, \( \theta \), in terms of the full-flow parameters \( k_0 \) and \( R^*_M \):

\[
Q_M^* = 4k_0^* R_M^* \sin \left( \frac{\theta}{2} \right) \sin \left( \frac{\theta}{2} \right)
\]

(14)

Comparing the Manning and DW equations requires specifying the full-flow relative roughness, \( k_0 \), and the full-flow Reynolds number, \( Re \). The non-dimensional parameters \( k_0 \) and \( Re \) are derived from the equivalent sand roughness, \( k_s \), pipe diameter, \( D \), and full-flow velocity, \( V \):

\[
A = \frac{2}{\sqrt{1 + \frac{k_s}{2} \frac{D}{h}}}
\]

and the corresponding normalized depth of flow, \( h^* \), in a pipe of diameter \( D \) is given by

\[
h^* = \frac{2}{D} \sqrt{1 - \cos \left( \frac{\theta}{2} \right)}
\]

(13)

where \( h \) is the actual flow depth. Equations 12 and 13 form a set of parametric equations that relate the normalized flow, \( Q_M^* \), to the normalized flow depth, \( h^* \), based on the DW equation.

If the Manning equation with constant \( n \) is used to calculate the flow rate as a function of depth, then the normalized Manning flow function is given by

\[
Q_M = A^* \left( h^* \right) R_M^{5/3} (15)
\]

and this equation is plotted in Figure 1 for various fixed values of \( Re \). The Manning equation is valid only for values of \( R^*_M \) greater than that given by Equation 17. Also shown in Figure 1 are the lines \( R^*_M = 4 k^*_0 \) and \( R^*_M = 500 k^*_0 \), and Manning's \( n \) can be taken as a constant only when \( R^*_M \) is between the two lines.

To illustrate the utility of Figure 1, suppose that full-flow conditions are such that \( Re = 10^5 \) and \( k^*_0 = 0.05 \). Then Manning's equation can only be used for \( R^*_M > 0.6 \), where it is noted that the condition for Manning's \( n \) being variable or constant requires fully turbulent flow as a precondition.

If full-flow conditions are such that \( Re = 5 \times 10^5 \) and \( k^*_0 = 0.05 \), then Figure 1 shows that the Manning equation can be used for \( R^*_M > 0.03 \), for a possible \( h^* \) must be used up to \( R^*_M = 0.2 \), and a constant \( n \) can be used for \( R^*_M > 0.2 \).

Since Figure 1 includes the practical ranges of \( k^*_0 \) and \( Re \), it is apparent that limitations can occur between the full-flow sewer capacity calculated using the Manning and DW equations, assuming the fully-turbulent relationship \( n = 0.039 k^*_0 \). Best agreement is achieved for higher values of the relative roughness, \( k_0 \), and agreement generally improves with increasing Reynolds number, \( Re \). For \( k^*_0 = 0.005 \) and \( Re = 10^5 \), the Manning full-flow capacity is on the order of 40 percent higher than the DW full-flow capacity—a result that is consistent with Figure 1, which indicates that the flow is not fully turbulent.

The discrepancy between \( Q_{Mfull} \) and \( Q_{Dfull} \) can be of significant concern in storm sewers that are designed under full-flow conditions using the Manning equation, since their actual...
Figure 4. Comparison between Manning and Darcy-Weisbach full-flow sewer capacities.

Figure 5. Comparison between Manning and Darcy-Weisbach flow velocities.

Continued from page 57

al capacity might be much less, resulting in higher flood frequency than their design specifications. Sanitary sewers are not designed to flow full, typically they are designed to flow either at 50 percent or 75 percent full, depending on the pipe diameter (Chin, 2006). For any given flow depth, the ratio of the flow rate using the Manning equation, QM, to the flow rate using the DW equation, QD, is

\[
\frac{Q_D}{Q_M} = \frac{Q_{D\text{full}}}{Q_{M\text{full}}} \quad (20)
\]

As shown previously, for cases where \( h^* > 0.22 \), it can be assumed that \( Q_{\text{Dfull}} \approx Q_{\text{Mfull}} \) with less than 5 percent error, and Equation 20 can be approximated by

\[
\frac{Q_D}{Q_M} \approx \frac{Q_{D\text{full}}}{Q_{D\text{full}}} \quad \text{when} \quad h^* > 0.22 \quad (21)
\]

Combining this result with \( Q_{\text{Dfull}}/Q_{\text{Mfull}} < 1 \) as a function of \( k_0 \) and \( Re \) given in Figure 4 demonstrates that sanitary sewers designed using the Manning equation to carry maximum flows at \( h^* \) equal to 50 percent or 75 percent will generally be under designed, resulting in higher flow depths and increasing the risk of sewer overflows.

Sewers are typically designed to achieve a minimum self-cleansing velocity of 0.6 m/s at the minimum flow rate. Under these flow conditions, it is likely that \( h^* < 0.22 \), and significant discrepancies between the Manning and Darcy-Weisbach flow functions can be expected.

Consider the case where the Manning equation is used in design, the normalized flow rate under minimum-flow conditions is \( Q_{\text{Mmin}} \), and the corresponding normalized velocity is \( V_{\text{Mmin}} \). For any given \( k_0 \) and \( Re \), the normalized flow rate using a DW design, \( Q_{\text{Dmin}} \), is related to \( Q_{\text{Mmin}} \) by

\[
\frac{Q_{\text{Dmin}}}{Q_{\text{Mmin}}} = \frac{Q_{D\text{full}}}{Q_{M\text{full}}} \quad (22)
\]

which corresponds to a normalized DW velocity \( V_{\text{Dmin}} \). The ratio of the actual minimum velocity computed using the Manning equation, \( V_{\text{Mmin}} \), to the actual minimum velocity computed using the DW equation, \( V_{\text{Dmin}} \), is then given by

\[
\frac{V_{\text{Mmin}}}{V_{\text{Dmin}}} = \frac{Q_{\text{Dmin}}}{Q_{\text{Mmin}}} \quad (23)
\]

The minimum-velocity ratio, \( V_{\text{Mmin}}/V_{\text{Dmin}} \), as a function of the Manning normalized minimum flow rate, \( Q_{\text{Mmin}}/Q_{\text{Mfull}} \), for \( k_0 = 0.0085 \) and \( Re = 10^4 \) are shown in Figure 5. It is apparent that for any specified minimum-flow condition, the actual minimum velocity calculated using the Manning equation, \( V_{\text{Mmin}} \), will be significantly greater than the actual minimum velocity calculated using the DW equation, \( V_{\text{Dmin}} \). In fact, for the case shown in Figure 5, the actual minimum-flow velocity might be on the order of one-half the self-cleansing velocity, even though a Manning design indicates that the self-cleansing velocity is achieved under minimum-flow conditions. Under this circumstance, sediment build-up is likely to be more of a problem than expected.

Achieving the self-cleansing velocity under minimum-flow conditions will vary with \( k_0 \) and \( Re \), but there is certainly cause for concern if the Manning equation is used to ensure a self-cleansing velocity.

Conclusions

The Manning equation is widely used in the design of sanitary and storm sewers in the United States, while usually little attention is given to verifying fully turbulent flow conditions and the assumption of a constant Manning’s n. For conditions that are typical in sewer design, fully turbulent flow conditions do not always occur and conditions for a constant Manning’s n do not always exist.

The Darcy-Weisbach (DW) equation is an attractive alternative to the Manning equation, since it does not depend on the flow regime. Parameters that must be specified in using the DW equation are the full-flow relative roughness, \( k_0 \), and the full-flow Reynolds number, \( Re \).

Significant discrepancies between the Manning and DW normalized flow functions are limited to flow depths that are less than 22 percent of the diameter. Significant discrepancies between the Manning and DW full-flow capacity can occur, especially for lower values of \( k_0 \) and \( Re \), where the fully turbulent criterion is likely to be violated.

Implications of the unconditional use of the Manning equation for design are that storm sewers might be sized too small to handle design flows under full-flow conditions, leading to excessive flooding; sanitary sewers might flow at depths greater than expected under maximum-flow conditions, resulting in increased frequencies of sewer overflows; and self-cleansing velocities might not be attained under minimum-flow conditions, leading to excessive sediment buildup.

References