Hydraulics of Sewer Flows

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Sanitary and storm sewers are an integral part of urban infrastructure, and their adequate design has a direct bearing on such undesirable consequences as sewer overflows and flooding. The hydraulic design of sewers depends fundamentally on the accuracy with which the flow rate can be expressed as a function of the depth of flow in the sewer.

The conventional approach to estimating the relationship between flow rate and flow depth in the United States is to use the Manning equation with a constant Manning's n. This approach is valid if the following conditions are met: (1) the flow is fully turbulent; and (2) the condition for a constant Manning's *n* is satisfied. In most cases of practical interest, these conditions are not met, the Manning equation is technically not applicable, and the Manning equation is used anyway. This article provides a clear delineation of the limitations of using the Manning equation to describe the hydraulics of sewer flows and provides an alternative formulation based on the Darcy-Weisbach equation.

Theory

Uniform flow in open channels under all flow regimes can be adequately described by the Darcy-Weisbach (DW) equation

$$Q = A\sqrt{\frac{8g}{f}}\sqrt{RS_0} \qquad (1)$$

where Q is the flow rate, A is the flow area, f is the friction factor, R is the hydraulic radius, and S_0 is the slope. The friction factor, f, can be approximated by (ASCE, 1963)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s}{12R} + \frac{2.5}{Re\sqrt{f}}\right) \qquad (2)$$

where k_s is the equivalent sand roughness and Re is the Reynolds number given by

$$Re = \frac{V(4R)}{\upsilon} \qquad (3)$$

where V is the average velocity (= Q/A) and _ is the kinematic viscosity of the fluid. Under turbulent-flow conditions, u^*k_s/_ > 70 (Yang, 1996; Rubin and Atkinson, 2001) where u^* = (gRS_0)1/2 and hence the turbulent flow criterion for water (_ = 1.0 10-6 m2/s) can be expressed as (Chin, 2006)

$$k_s\sqrt{RS_0} > 2.2\times10^{-5} \qquad (4)$$

where S_0 is the slope of the channel. For water under fully turbulent flow conditions, the Manning equation can be used in lieu of the Darcy-Weisbach equation, in which case,

$$Q = A\frac{R^{1/6}}{n}\sqrt{RS_0} \qquad (5)$$

where *n* is the Manning roughness coefficient. Comparing the Manning equation with the Darcy-Weisbach equation for fully turbulent flow conditions, (Re _ ∞) gives the following expression for the variation of *n* as a function of hydraulic radius:

$$\frac{n}{k_s^{1/6}} = \frac{\frac{1}{\sqrt{8g}}\left(\frac{R}{k_s}\right)^{1/6}}{2\log\left(\frac{12R}{k_s}\right)} \qquad (6)$$

The right-hand side of this equation remains approximately constant (± 5 percent) when

$$4 < \frac{R}{k_s} < 500 \qquad (7)$$

in which case (Sturm, 2001)

$$\frac{n}{k_s^{1/6}} = 0.039 \qquad (8)$$

The conventional method of estimating the flow rate in sewers as a function of the flow depth is using the Manning equation with constant n, which requires that both Equations 4 and 7 are satisfied. The Manning equation can be used with variable *n* provided that Equation 4 is satisfied and the variability in *n* as a function of flow depth is described by Equation 6. In contrast to these limitations on the Manning equation, the DW equation can be used to describe the flow rate as a function of depth of flow for all regimes.

Analysis

It is convenient to describe sewer hydraulics in terms of variables that are normalized relative to full-flow values. Denoting normalized quantities by asterisks, the DW equation (Equations 1 and 2) can be expressed in the form

$$Q_D^* = A^*\sqrt{R^*}\,\frac{\log\left[\frac{k_0}{12}\left(\frac{1}{R^*}\right) + \frac{2.5}{Re_0\sqrt{f_0}}\left(\frac{1}{R^{*3/2}}\right)\right]}{\log\left[\frac{k_0}{12} + \frac{2.5}{Re_0\sqrt{f_0}}\right]} \qquad (9)$$

where Q_D^* is the flow rate normalized relative to the full-flow flow rate, k_0 is the full-flow relative roughness, and Re_0 is the full-flow Reynolds number such that

$$k_0 = \frac{k_s}{R_0} \quad \text{and} \quad Re_0 = \frac{V_0(4R_0)}{\upsilon} \qquad (10)$$

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Table 1. Typical Ranges of Design Parameters for Sanitary Sewers

Design Parameters			Derived Parameters	
k_s (mm)	D (mm)	V_0 (m/s)	k_0 (dimensionless)	Re_0 (dimensionless)
0.51 – 3.24	305 – 2745	0.6 – 3.5	7.4×10^{-4} – 0.0425	1.83×10^5 – 9.61×10^6

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where R0 is the full-flow hydraulic radius, V_0 is the full-flow velocity, and f_0 is the full-flow friction factor corresponding to k_0 and Re_0 as given by Equation 2 for full-flow conditions. The geometric quantities A* and R* can be expressed as

$$A^*(\theta) = \frac{\theta - \sin\theta}{2\pi} \quad \text{and} \quad R^*(\theta) = \frac{\theta - \sin\theta}{\theta} \tag{11}$$

where _ is the apex angle of the triangle formed by the center of the pipe (apex) and the top width of the water surface (base). Combining Equations 9 and 11 gives the following relationship between the normalized flow rate, Q_D*, and apex angle, _, in terms of the full-flow parameters k_0 and Re_0,

$$Q_D^* = \frac{\theta R^{*3/2}(\theta)}{2\pi} \frac{\log\left[\frac{k_0}{12}\left(\frac{1}{R^*(\theta)}\right) + \frac{2.5}{Re_0\sqrt{f_0}}\left(\frac{1}{R^{*3/2}(\theta)}\right)\right]}{\log\left[\frac{k_0}{12} + \frac{2.5}{Re_0\sqrt{f_0}}\right]} \tag{12}$$

and the corresponding normalized depth of flow, h*, in a pipe of diameter D is given by

$$h^* = \frac{h}{D} = \frac{1}{2}\left[1 - \cos\left(\frac{\theta}{2}\right)\right] \tag{13}$$

Where h is the actual flow depth. Equations 12 and 13 form a set of parametric equations that relate the normalized flow, Q_D*, to the normalized flow depth, h*, based on the DW equation.

If the Manning equation with constant *n* is used to calculate the flow rate as a function of depth, then the normalized Manning flow function is given by

$$Q_M^* = A^*(\theta)R^{*2/3}(\theta) \tag{14}$$

Comparing the Manning and DW equations requires specifying the full-flow relative roughness, k_0, and the full-flow Reynolds number, Re_0. The non-dimensional parameters k_0 and Re_0 are derived from the equivalent sand roughness, ks, pipe diameter, D, and full-flow velocity, V_0.

Typical values of k_s for concrete pipes can be estimated using the turbulent-flow relationship between *n* and k_s given by Equation 8, where typical *n* values of 0.011 to 0.015 correspond to k_s values of 0.51 mm to 3.24 mm. Typical pipe diameters, D, are the commercial pipe sizes between 305 mm and 2,745 mm, and typical full-flow velocities are in the range of 0.6 m/s to 3.5 m/s (Chin, 2006). These ranges are given in Table 1, along with the corresponding ranges of the full-flow parameters k_0 and Re_0.

The criterion for fully-turbulent flow (Equation 4) can be expressed in terms of the full-flow parameters k_0 and Re_0 by

$$R^* > \frac{6.08 \times 10^5}{(k_0 Re_0)^2 f_0} \tag{15}$$

where f_0 is the full-flow friction factor corresponding to k_0 and Re_0 as given by Equation 2 for full-flow conditions. In cases where fully turbulent flow exists, the condition for a constant Manning's *n* is given by Equation 7, which can be expressed as

$$4k_0 < R^* < 500k_0 \tag{16}$$

For any given values of k_0 and Re_0, the limiting condition of fully turbulent flow is

$$R^* = \frac{6.08 \times 10^5}{(k_0 Re_0)^2 f_0} \tag{17}$$

and this equation is plotted in Figure 1 for various fixed values of Re_0. The Manning equation is valid only for values of R* greater than that given by Equation 17. Also shown in Figure 1 are the lines R* = 4 k_0 and R* = 500 k_0, and Manning's *n* can be taken as a constant only when R* is between the two lines.

To illustrate the utility of Figure 1, suppose that full-flow conditions are such that $Re_0 = 10^5$ and $k_0 = 0.05$. Then Manning's equation can only be used for R* > 0.6, where it is noted that the condition for Manning's *n* being variable or constant requires fully turbulent flow as a precondition.

If full-flow conditions are such that $Re_0 = 5 \times 10^5$ and $k_0 = 0.05$, then Figure 1 shows that the Manning equation can be used for R* > 0.03; however, a variable *n* must be used up to R* = 0.2, and a constant *n* can be used for R* > 0.2.

Since Figure 1 includes the practical ranges of k_0 and Re_0, it is apparent that limitations in using the Manning equation are likely to be encountered in practice, and there is no case where the Manning equation is valid for all flow depths.

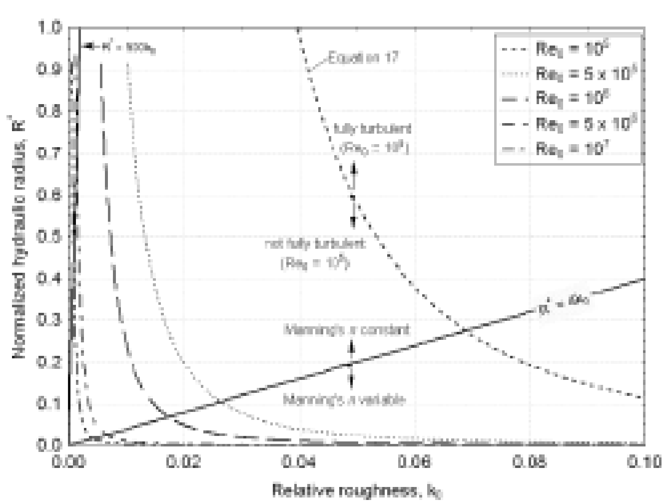


Figure 1. Conditions for the validity of the Manning equation.

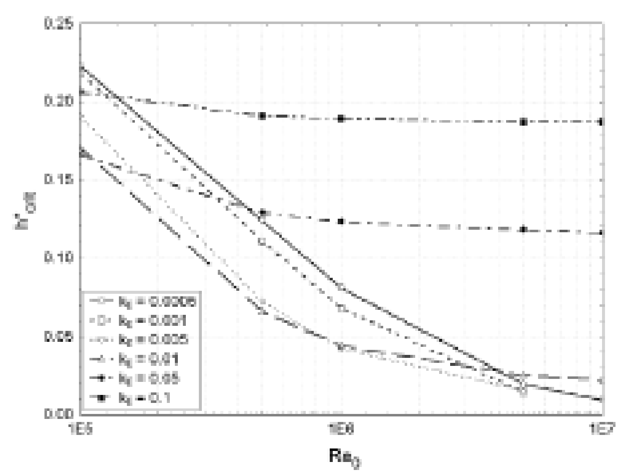


Figure 2. Critical flow depth for the difference between Manning and Darcy-Weisbach equations to be less than 5 percent.

The flow functions Q_D*(h*), given by Equation 12, and Q_M*(h*), given by Equation 14, were compared for the practical cases of $0.0005 < k_0 < 0.1$ and $10^5 < Re_0 < 10^7$, and differences were assessed in terms of percentage difference, EM/D, at each flow depth, h*, as given by

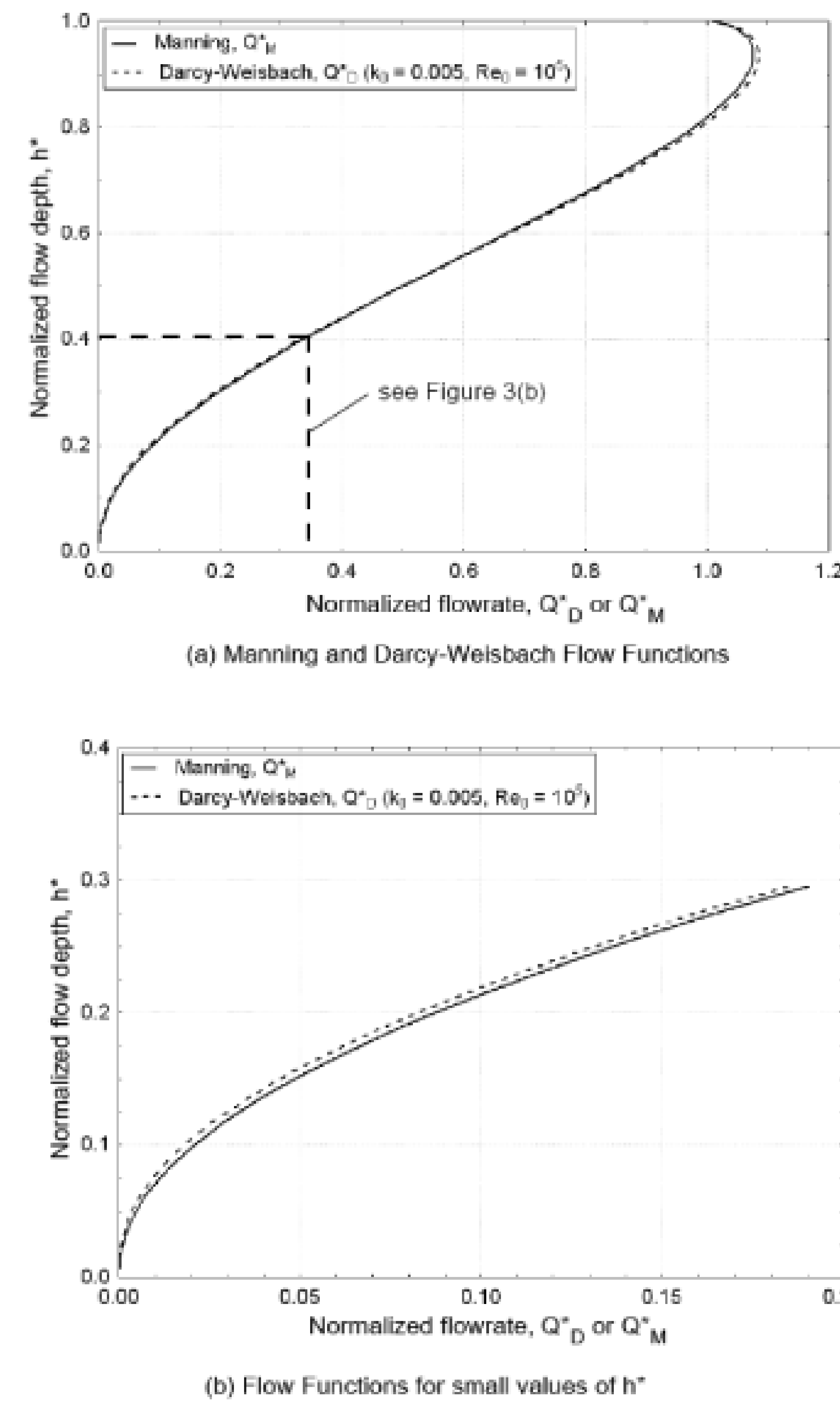
$$E_{M/D}(h^*) = \frac{Q_M^*(h^*) - Q_D^*(h^*)}{Q_D^*(h^*)} \tag{18}$$

In all practical cases, the highest percentage differences occur at low values of h*, and it is convenient to define a critical flow depth, h*crit, above which the difference, EM/D(h*), is less than 5 percent.

Values of h*crit as a function of Re_0 and k_0 are shown in Figure 2, which indicates that the difference between Q_D*(h*) and Q_M*(h*) generally does not exceed 5 percent for h* > 0.22. Figure 2 also indicates that for given values of k_0, h*crit typically decreases as Re_0 increases, and for given values of Re_0, h*crit depends nonlinearly on k_0.

The maximum value of h*crit occurs when $k_0 = 0.0005$ and $Re_0 = 10^5$, and the corresponding flow functions Q_D*(h*) and Q_M*(h*) for these values of k_0 and Re_0 are shown in Figure 3(a). The Manning and DW flow functions appear to be in close agreement, with small differences occurring at low and high values of h*.

Figure 3. Comparison between Manning and Darcy-Weisbach flow functions.



Discrepancies at low values of h* can correspond to high percentage errors, as is apparent from the close-up view shown in Figure 3(b). These discrepancies at low values of h* can be particularly important when assessing sewer performance under low-flow conditions when self-cleansing velocities must be attained.

Close agreement between the normalized flow functions Q_D*(h*) and Q_M*(h*) does not necessarily mean that the actual flow distributions are in close agreement, since Q_M*(h*) is normalized relative to the full-flow Manning discharge, Q_{Mfull}, and Q_D*(h*) is normalized relative to the full-flow DW discharge, Q_{Dfull}. Taking *n* = 0.039ks1/2, as given by Equation 8, the ratio of the Manning full-flow discharge to the DW full-flow discharge can be expressed in terms of k_0 and Re_0 as

$$\frac{Q_{Mfull}}{Q_{Dfull}} = \frac{A_0 \frac{R_0^{1/6}}{n}\sqrt{R_0 S_0}}{A_0\sqrt{\frac{8g}{f_0}}\sqrt{R_0 S_0}} = 2.894\frac{\sqrt{f_0}}{k_0^{1/6}} \tag{19}$$

where f_0 is a function of k_0 and Re_0 as given by Equation 2 for full-flow conditions.

Values of Q_{Mfull}/Q_{Dfull} for $0.0005 < k_0 < 0.1$ and $10^5 < Re_0 < 10^7$ are shown in Figure 4, where it is apparent that significant deviations can occur between the full-flow sewer capacity calculated using the Manning and DW equations, assuming the fully-turbulent relationship $n = 0.039k_s^{1/6}$. Best agreement is achieved for higher values of the relative roughness, k_0, and agreement generally improves with increasing Reynolds number, Re_0. For $k_0 = 0.0005$ and $Re_0 = 10^5$, the Manning full-flow capacity is on the order of 40 percent higher than the DW full-flow capacity—a result that is consistent with Figure 1, which indicates that the flow is not fully turbulent.

The discrepancy between Q_{Mfull} and Q_{Dfull} can be of significant concern in storm sewers that are designed under full-flow conditions using the Manning equation, since their actu-

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Figure 4. Comparison between Manning and Darcy-Weisbach full-flow sewer capacities.

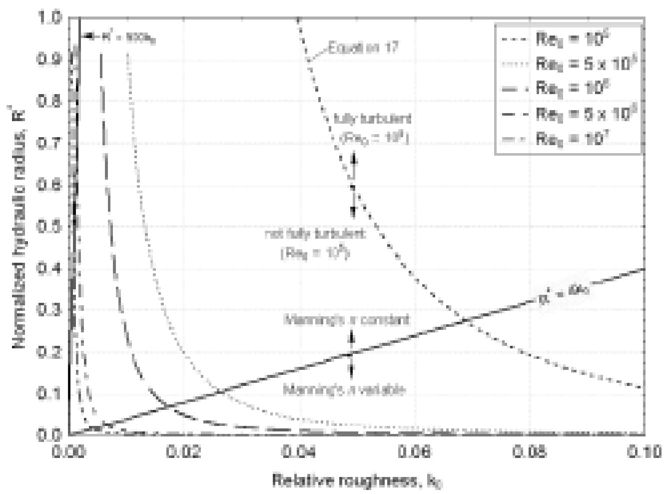
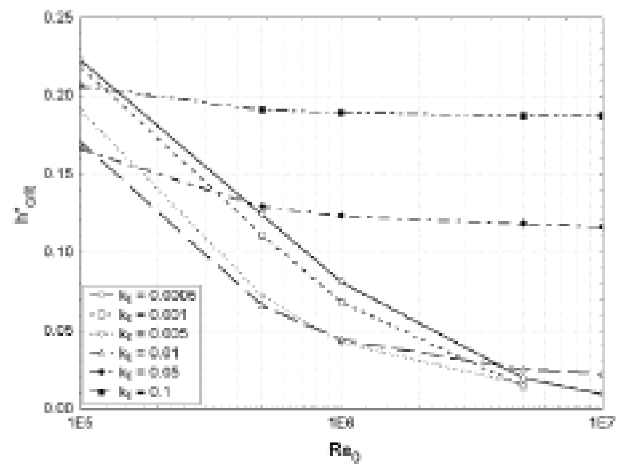


Figure 5. Comparison between Manning and Darcy-Weisbach flow velocities.



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al capacity might be much less, resulting in higher flood frequency than their design specifications. Sanitary sewers are not designed to flow full; typically they are designed to flow either 50 percent or 75 percent full, depending on the pipe diameter (Chin, 2006). For any given flow depth, the ratio of the flow rate using the Manning equation, QM, to the flow rate using the DW equation, Q_D, is

$$\frac{Q_M}{Q_D} = \frac{Q_M^*}{Q_D^*} \times \frac{Q_{Mfull}}{Q_{Dfull}} \qquad (20)$$

As shown previously, for cases where h* > 0.22, it can be assumed that $Q_M^* \approx Q_D^*$ with less than 5 percent error, and Equation 20 can be approximated by

$$\frac{Q_M}{Q_D} \approx \frac{Q_{Mfull}}{Q_{Dfull}} \quad \text{when} \quad h^* > 0.22 \qquad (21)$$

Combining this result with Q_{Mfull}/Q_{Dfull} as a function of k_0 and Re_0 given in Figure 4 demonstrates that sanitary sewers designed using the Manning equation to carry maximum flows at h* equal to 50 percent or 75 percent will generally be under designed, resulting in higher flow depths and increasing the risk of sewer overflows.

Sewers are typically designed to achieve a minimum self-cleansing velocity of 0.6 m/s at the minimum flow rate. Under these flow conditions, it is likely that h* < 0.22, and significant discrepancies between the Manning and Darcy-Weisbach flow functions can be expected.

Consider the case where the Manning equation is used in design, the normalized flow rate under minimum-flow conditions is Q_{Mmin}^*, and the corresponding normalized velocity is V_{Mmin}^*. For any given k_0 and Re_0, the normalized flow rate using a DW design, Q_{Dmin}^*, is related to Q_{Mmin}^* by

$$\frac{V_{M\,min}}{V_{D\,min}} = \frac{V_{M\,min}^*}{V_{D\,min}^*} \times \frac{Q_{Mfull}}{Q_{Dfull}} \qquad (22)$$

which corresponds to a normalized DW velocity V_{Dmin}^*. The ratio of the actual minimum velocity computed using the Manning equation, V_{Mmin}, to the actual minimum velocity computed using the DW equation, V_{Dmin}, is then given by

$$Q_{D\,min}^* = Q_{M\,min}^* \times \frac{Q_{Mfull}}{Q_{Dfull}} \qquad (23)$$

The minimum-velocity ratio, V_{Mmin}/V_{Dmin} as a function of the Manning normalized minimum flow rate, Q_{Mmin}^*, for $k_0 = 0.0005$ and $Re_0 = 10^5$ are shown in Figure 5. It is apparent that for any specified minimum-flow condition, the actual minimum velocity calculated using the Manning equation, V_{Mmin}, will be significantly greater than the actual minimum velocity calculated using the DW equation, V_{Dmin}. In fact, for the case shown in Figure 5, the actual minimum-flow velocity might be on the order of one-half the self-cleansing velocity, even though a Manning design indicates that the self-cleansing velocity is achieved under minimum-flow conditions. Under this circumstance, sediment build-up is likely to be more of a problem than expected.

Achieving the self-cleansing velocity under minimum-flow conditions will vary with k_0 and Re_0, but there is certainly cause for concern if the Manning equation is used to ensure a self-cleansing velocity.

Conclusions

The Manning equation is widely used in the design of sanitary and storm sewers in the United States, while usually little attention is given to verifying fully turbulent flow conditions and the assumption of a constant Manning's n. For conditions that are typical in sewer design, fully turbulent flow conditions do not always occur and conditions for a constant Manning's *n* do not always exist.

The Darcy-Weisbach (DW) equation is an attractive alternative to the Manning equation, since it does not depend on the flow regime. Parameters that must be specified in using the DW equation are the full-flow relative roughness, k_0, and the full-flow Reynolds number, Re_0.

Significant discrepancies between the Manning and DW normalized flow functions are limited to flow depths that are less than 22 percent of the diameter. Significant discrepancies between the Manning and DW full-flow capacity can occur, especially for lower values of k_0 and Re_0 where the fully turbulent criterion is likely to be violated.

Implications of the unconditional use of the Manning equation for design are that storm sewers might be sized too small to handle design flows under full-flow conditions, leading to excess flooding; sanitary sewers might flow at depths greater than expected under maximum-flow conditions, leading to increased frequencies of sewer overflows; and self-cleansing velocities might not be attained under minimum-flow conditions, leading to excessive sediment buildup.

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